Surface Pressure and Streamline Effects on Laminar Heating Calculations

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The effect of streamline geometry and pressure distributions on surface heating rates is examined for slender, spherically blunted cones. The resulting modifications to an approximate aeroheating code include a curve fit of pressures computed by an Euler solution over a range of Mach numbers and small cone angles. An existing correlation based on pressures computed by the method of characteristics is used for larger cone angles. The streamline geometry is computed using the surface pressures and inviscid surface properties. Streamline calculations based on inviscid surface conditions rather than boundary-layer edge properties are demonstrated to yield improved heating analyses. However, the heating rates are calculated using inviscid properties at the boundary-layer edge. Resulting heating rates compare favorably with solutions from the viscous-shock-layer equations.

Nomenclature

c,d,e	= coefficients used in curve fit of surface pressures
f	= body radius, measured from longitudinal axis
h	= scale factor (metric) in β direction
M	= Mach number
p	= pressure
$\dot{q}$	= heat-transfer rate
R_N	= nose radius of body
S	= distance along a streamline measured from stagnation point
T	= Chebyshev polynomials
V	= velocity
x,y,z	= Cartesian coordinates
α	= angle of attack
β	= coordinate normal to streamline on body surface
$\Gamma,\delta_\phi,\sigma$	= body angle depicted in Fig. 2
θ	= inclination angle of surface streamline
ρ	= density
τ	= combination of Chebyshev polynomials
ϕ	= cylindrical coordinate (see Fig. 2)
ω	= inclination angle of surface with respect to x axis, rad

Subscripts

c	= cone value
eq	= equivalent
i	= value at interface of spherical cap and conical afterbody
ref	= reference condition
stg	= stagnation condition
w	= wall value
∞	= freestream condition

Superscript

$'$	= normalized variable used in curve fit of surface pressures

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Introduction

RECENT interest in hypersonic vehicles has resulted in a re-examination of both detailed and engineering flowfield methods. The engineering methods employ various levels of approximation to calculate flowfields, shock shapes, and pressure distributions as well as the surface heating rates. In addition, they require far less CPU time than solutions of the full flowfield equations. For this reason, approximate methods are ideal for preliminary and parametric studies of re-entry vehicles.

DeJarnette and Hamilton,[1] in the early 1970's, developed an approximate method to calculate heating rates on three-dimensional configurations at angle of attack. The method involves tracing inviscid surface streamlines from the stagnation region and the "axisymmetric analog" concept[2] is used so that axisymmetric boundary-layer methods can be applied to three-dimensional flows. Modifications[3] to the code were made later to account for entropy-layer swallowing.

Thompson et al.[4] examined one version of the code, which is called AEROHEAT or NHEAT, and several other engineering methods used to calculate surface heating rates. One result from that investigation was that the engineering codes did not predict satisfactory results for slender cones at small angles of attack, e.g., a blunt 5-deg cone at 3-deg angle of attack. However, the engineering codes have been shown to yield good comparisons with experimental heating data on blunter cones at angle of attack.[5] Another result noted in the investigation[4] was that the AEROHEAT code assumed a Newtonian pressure distribution and the corresponding predicted heating rates even for 0-deg angle-of-attack (AOA) conditions did not yield good comparison with detailed solutions over slender configurations. Since this engineering code is used throughout the aerospace community, the deficiencies noted in the investigation of Ref. 4 need to be resolved. Thus, the purpose of this paper is to discuss recent and pertinent modifications to the AEROHEAT code, which was developed by DeJarnette. The applicability of the Newtonian pressure distribution for heating calculations over other vehicle configurations (e.g., relatively flat bottom vehicles) and conditions is not addressed in this study. However, the study of Ref. 4 for slender blunted cones and the results presented in Ref. 1 for a shuttle configuration suggest that the user should exercise caution when employing the AEROHEAT code.

The paper presents a new pressure correlation for slender, spherically blunted cones, and discusses modifications for the streamline and metric calculations. The impact of these changes on the heating calculations is illustrated by comparison with results of a detailed code.

Analysis

Detailed Code

In this study, the present heating results of the AEROHEAT code are compared to detailed predictions of a viscous-shock-layer (VSL) code[4] to assess the effects of the present modifications. The VSL method is based on a solution of a subset of the Navier-Stokes equations in which parabolic approximations are made in both the streamwise and crossflow directions and terms up to second order in the inverse square root of the Reynolds number are retained. This set of governing equations is uniformly valid throughout the shock layer and directly accounts for the interaction between the inviscid and viscous regions due to heat transfer, entropy-layer swallowing, and mass injection. The VSL method has been applied to planetary entry bodies with massive ablation and radiation,[6] to slender bodies with transitional and turbulent flow,[7] and to complex re-entry vehicles with nonequilibrium flowfields.[8] In the study of Ref. 4, the results of a three-dimensional VSL code showed very good agreement when compared to flight- and ground-test laminar and turbulent heat-transfer data and to results of other detailed code predictions.

Engineering Code

Heating rates are calculated in AEROHEAT in the following manner. First, inviscid streamlines are traced from the stagnation region along the body surface. These streamlines form a curvilinear streamwise coordinate system. Next, the boundary-layer properties are calculated along the streamlines based on the surface pressure and the local entropy. Two options are available for determining the edge entropy. The first, and simplest, uses normal-shock entropy whereas the second option accounts for variable-entropy edge conditions. In this option, a procedure based on mass balancing determines the bow-shock crossing location of the boundary-layer edge streamline. These postshock conditions are characterized by different entropy values. Thus, entropy-layer swallowing accounts for the entrainment of the inviscid streamlines by the growing boundary layer (Fig. 1).

The axisymmetric analog concept is now applied with the crossflow (flow tangent to the surface and normal to the streamlines) assumed to be zero. This concept allows the three-dimensional boundary-layer equations to be reduced to the axisymmetric form provided the distance along the streamline is substituted for the wetted surface distance and the metric coefficient describing the divergence of the streamlines is interpreted as the cross-sectional body radius. Heating-rate prediction methods that are applicable to axisymmetric flows can then be employed.

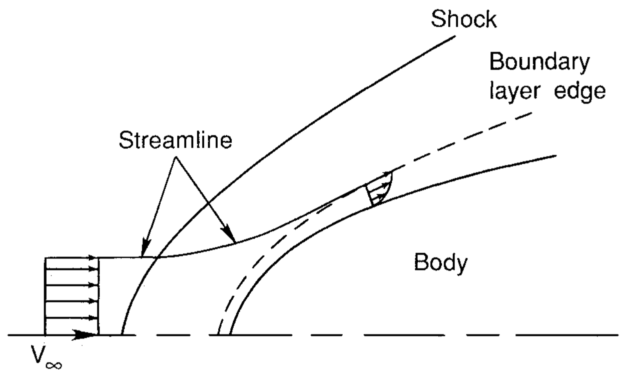


Fig. 1 Entropy-layer swallowing.

Pressure Distribution

Surface pressures are presently computed in AEROHEAT using modified Newtonian theory. Although this method is simple to apply and reasonably accurate for some geometries, it does not adequately predict the pressure on spherically blunted, slender cones. The investigation of Ref. 4 demonstrated that the corresponding heating-rate predictions were only in fair agreement with VSL solutions even at 0-deg AOA conditions.

For this reason, pressures obtained from the finite-difference solution of the Euler equations for the flow over a blunt cone[9] were correlated and curve fit. The correlation included cone half-angles of 5–10 deg and Mach numbers from 5 to 25 for an angle of attack of zero. For cone angles larger than 10 deg, the pressure correlation of Ref. 10 based on pressures computed by the method of characteristics is used. The present curve fit is a fifth-order polynomial fit using Chebyshev polynomials. First, the axial distance, cone half-angle, and freestream Mach numbers are normalized as follows:

$$x' = \frac{2\ln(x/R_N\omega_c^2) + 4.6188}{5.3188}$$

$$[(x/R_N)\omega_c^2]_i \le [(x/R_N)\omega_c^2] \le 1.42 \tag{1}$$

$$\omega_c' = \frac{2\omega_c - 0.2618}{0.08727}, \quad 0.08727 \le \omega_c \le 0.1745 \tag{2}$$

$$M_\infty' = \frac{2M_\infty - 30}{20}, \quad 5 \le M_\infty \le 25 \tag{3}$$

The pressure is then given by

$$\ln\left(\frac{p}{p_c}\right) = \sum_{j=2}^{5} c_j \tau_j(x') \tag{4}$$

where τ_j involves linear combinations of the Chebyshev polynomials of x' as

$$\tau_j(x') = (j^2 - 1)T_0(x') - j^2 T_1(x') + T_j(x')$$

$$2 \le j \le 5 \tag{5}$$

where the various T are the Chebyshev polynomials and are defined by

$$T_0(x') = 1 \tag{6a}$$

$$T_1(x') = x' \tag{6b}$$

$$T_j(x') = 2x' T_{j-1} - T_{j-2}, \qquad j \geq 2 \tag{6c}$$

The coefficients of τ_j vary with respect to ω_c' and M_c' and are determined from the following relations:

$$c_i = \sum_{j=0}^{2} d_{ij} T_j(\omega_c'), \qquad 2 \leq i \leq 5 \tag{7a}$$

$$d_{ij} = \sum_{k=0}^{3} e_{ijk} T_k(M_\infty'), \qquad 2 \leq i \leq 5, \qquad 0 \leq j \leq 2 \tag{7b}$$

The matrix e is given in Table 1. For $[(x/R_N)\omega_c^2] > 1.42$ the pressure is assumed to be that of a sharp cone.

The surface pressure on the spherical cap of the blunted cone is found by coupling modified Newtonian pressures with a blending function such that the pressure is continuous at the interface of the spherical cap and conical afterbody. The pressure is given by

$$p = p_{\text{stg}} \sin^2\omega + p_\infty \cos^2\omega + (p_i - p_{\text{stg}} \sin^2\omega_c - p_\infty \cos^2\omega_c)\frac{\cos^2\omega}{\cos^2\omega_c} \tag{8}$$

For an AOA condition, the rotated body approximation is used to determine the pressure distribution. An "equivalent body angle" is defined as the body angle relative to the freestream velocity vector. For an axisymmetric body at AOA, this is equal to

$$\omega_{\text{eq}} = \sin^{-1}(\cos\alpha \sin\omega + \sin\alpha \cos\omega \cos\phi)$$

where ϕ is the angle in the circumferential direction. In the windward plane of a cone at AOA, the equivalent body angle is equal to $(\omega + \alpha)$. Thus, the pressure distribution for an "equivalent" blunted cone at 0-deg AOA is used to approximate the AOA surface pressures. Derivatives of the pressure are easily found in the plane of symmetry using the pressure correlation for the equivalent body. However, out of the plane of symmetry, the derivative $\partial p/\partial\phi$ must be numerically approximated by using the pressure curve fits for the equivalent body angles at several ϕ locations. In this manner, the pressure and its derivatives are obtained over the entire spherically blunted cone.

Table 1 Pressure correlation coefficient matrix e

i	j	k 0	1	2	3
2	0	0.3600	0.3187	−0.04540	−0.007634
	1	0.04782	0.2666	−0.008651	−0.02518
	2	0.02747	0.03680	−0.01691	0.004930
3	0	0.09437	−0.08594	−0.02093	0.02056
	1	−0.06106	−0.2312	0.01483	0.01792
	2	−0.02699	−0.009968	0.01416	−0.005674
4	0	0.01471	0.1390	0.001666	−0.01518
	1	0.08810	0.1212	−0.02379	−0.002535
	2	0.004867	−0.01108	−0.005131	0.003860
5	0	−0.07211	−0.08659	0.009207	0.003692
	1	−0.03986	−0.02592	0.01090	−0.001803
	2	0.002774	0.007046	0.0002624	−0.01111

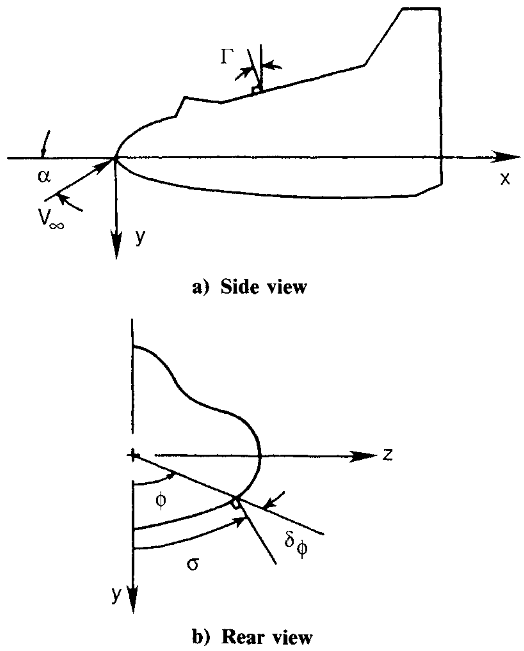


Fig. 2 Geometry of a general three-dimensional body.

Inviscid Streamline Calculations

One version of AEROHEAT uses the method of steepest descent to calculate the inviscid surface streamlines, and the results are referred to as simplified or Newtonian streamlines.[11] The direction of these streamlines is given by the resultant of the freestream velocity vector minus its component normal to the surface. Moreover, since the streamlines are found independently of the surface pressure distribution, the method is relatively easy to apply to any geometry.

An alternate method[1] for calculating the surface streamlines from a known surface pressure distribution was also developed. The Euler equation was applied to obtain the following set of differential equations for the inviscid surface streamlines and metric. (See Fig. 2 for description of angles.)

$$\begin{aligned}\frac{D\theta}{DS} = &-\left(\frac{1}{\rho V^2}\right)\left[-\sin\theta \cos\Gamma \frac{\partial p}{\partial x} + \frac{(\cos^1\theta \cos\delta_\phi + \sin\theta \sin\delta_\phi \sin\Gamma)}{f}\frac{\partial p}{\partial \phi}\right] \\ &- \sin\Gamma\left[\cos\theta \cos\Gamma \frac{\partial \sigma}{\partial x} + \frac{(\sin\theta \cos\delta_\phi - \cos\theta \sin\delta_\phi \sin\Gamma)}{f}\frac{\partial \sigma}{\partial \phi}\right]\end{aligned} \tag{9}$$

$$\frac{Dx}{DS} = \cos\theta \cos\Gamma \tag{10}$$

$$\frac{D\phi}{DS} = \frac{\sin\theta \cos\delta_\phi - \cos\theta \sin\delta_\phi \sin\Gamma}{f} \tag{11}$$

$$\begin{aligned}\frac{1}{h}\frac{D^2 h}{DS^2} = &-\left(\frac{1}{\rho V^2}\frac{1}{h}\frac{\partial p}{\partial \beta}\right)^2 (3 - M^2) \\ &- \frac{1}{\rho V^2}\frac{1}{h}\frac{\partial}{\partial \beta}\left(\frac{1}{h}\frac{\partial p}{\partial \beta}\right) \\ &+ \frac{\cos^2\Gamma \cos\delta_\phi}{f}\left(\frac{\partial \Gamma}{\partial x}\frac{\partial \sigma}{\partial \phi} - \frac{\partial \sigma}{\partial x}\frac{\partial \Gamma}{\partial \phi}\right)\end{aligned} \tag{12}$$

The preceding set of equations determines the streamline coordinate system along the body. Note that a density, velocity, and Mach number appear in the equations. For the variable-entropy heating option in the code, the present method for computing the streamlines and metrics[3] uses the density and velocity at the boundary-layer edge in these equations as well as in the heating-rate equations. Previous applications of AEROHEAT were at ground-test conditions where the effects of variable-entropy edge conditions on laminar heating rates are small, and the applicability of that method for calculating the streamlines and metrics was not assessed. However, this technique resulted in lower computed heating rates at variable-entropy conditions than at the corresponding normal-shock entropy conditions. These calculations were presented in Ref. 3 for blunted 15- and 7.2-deg cones at angle of attack (Figs. 6 and 7, respectively, of that reference). Intuitively, the opposite trend should have been expected, but since the study of Ref. 12 produced results similar to those of Ref. 3, the trend was attributed to influence of angle of attack on the streamlines. More recent analyses[13,14] use the inviscid surface properties in the calculation of the streamline and the metric, and the expected trends are noted.

After reviewing the procedure[3] to calculate the streamlines and metrics, the use of boundary-layer edge density, velocity, and Mach number is determined to be incompatible with the calculation procedure. The use of these properties results in computed coordinates that are not the surface streamline coordinates but correspond to a series of different streamlines as the equations are integrated over the vehicle. The density, velocity, and Mach number in the surface streamline equations should be based on normal-shock entropy conditions for proper inviscid surface streamlines to be computed. However, variable-entropy edge conditions should be used in the heat-transfer calculations.

Results and Discussion

All heating rates were calculated using surface pressures and variable-entropy edge conditions. Surface pressures are based on either modified Newtonian theory or the present pressure correlations. Streamlines and metrics were computed using the following: geometry only (simplified streamlines), surface pressures and normal-shock entropy, or surface pressures and variable-entropy edge conditions.

Computed pressure and heat-transfer results are presented for a 5-deg spherically blunted cone at angles of attack of 0, 3, and 20 deg at a Mach number of 15 for laminar flow of a perfect gas. Freestream conditions are for an altitude of 150,000 ft and the nose radius of the blunted cone is 0.125 ft. The ratio of the wall enthalpy to the adiabatic wall enthalpy is 0.1. All pressures and heating rates presented in Figs. 3–6 are computed along the windward ray to illustrate the improved results that are computed with the present pressure correlations and the modified streamline calculation procedure. Comparisons are made using both the modified Newtonian pressure and the inviscid pressure correlations in a revised AEROHEAT. Solutions from the VSL code are used as a benchmark. For the sake of clarity, the various methods for computing the pressure distributions and streamline paths are identified by initials in the figures. These methods with corresponding initials are defined in Table 2. The windward symmetry-plane pressure distributions computed for the 5-deg sphere cone at $M_\infty = 15$ for the three angles of attack are presented in Fig. 3. Note the modified Newtonian theory does not predict the pressure levels in the overexpansion/recompression region adequately. The results of the pressure correlations with the rotated body approximations for the AOA cases are shown to yield good comparison, as might be expected, with the results of the detailed inviscid code. The computed VSL pressures are presented in Fig. 3a to illustrate the magnitude of viscous effects on the pressures. The differences in the viscous and in-

Table 2 AEROHEAT options

Pressure distribution	Modified Newtonian pressures
	Pressure correlation
Streamline calculation procedure	Simplified streamlines
	Uses inviscid surface properties (from normal-shock entropy)
	Uses inviscid boundary-layer edge properties (from variable entropy)

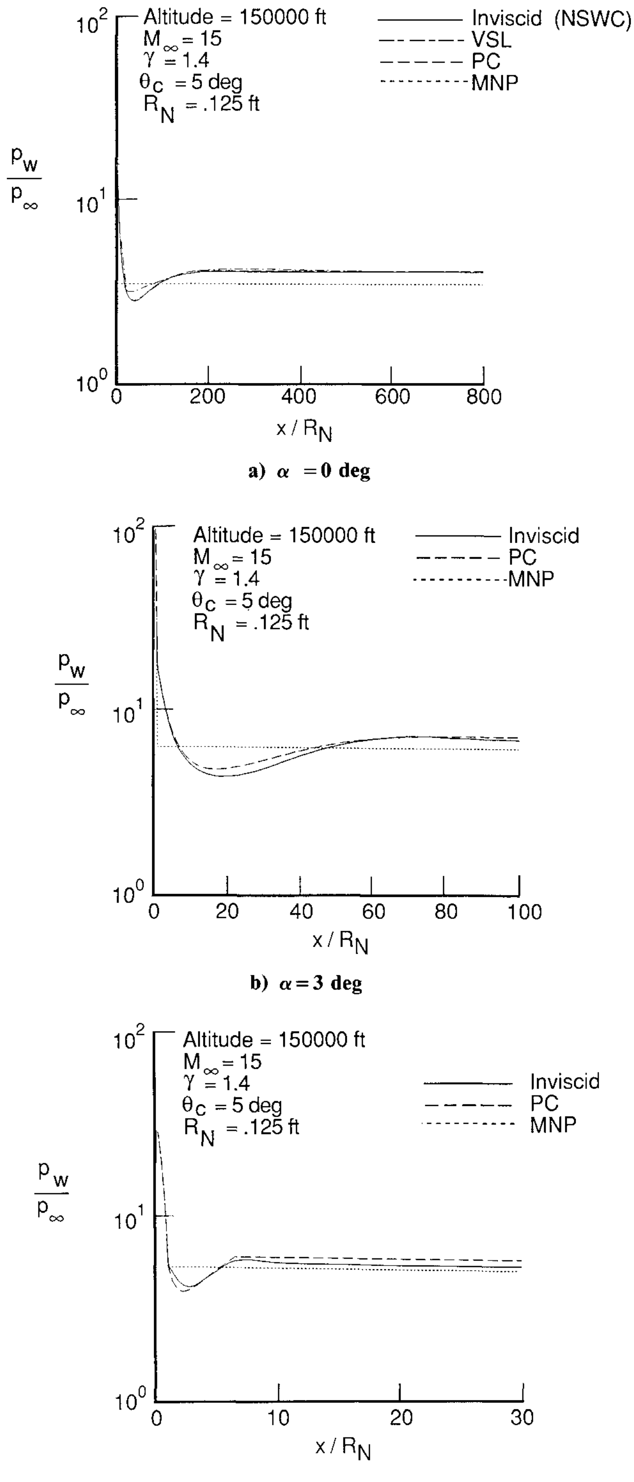


Fig. 3 Surface pressure distribution along windward ray.

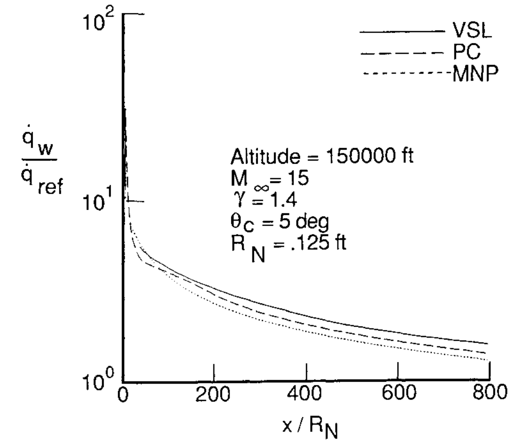
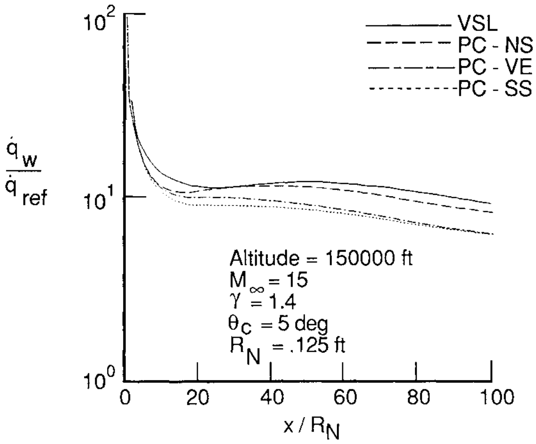
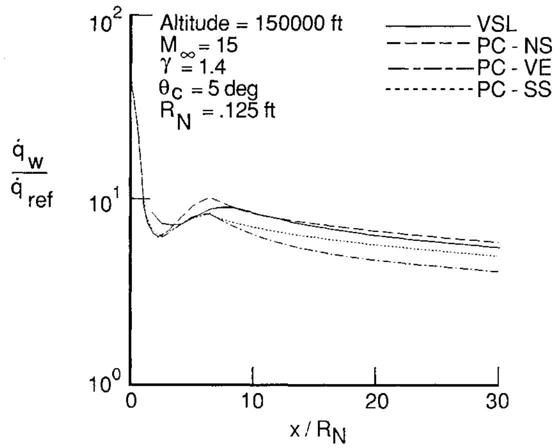


Fig. 4 Comparison of surface heating-rate predictions at $\alpha = 0$ deg.

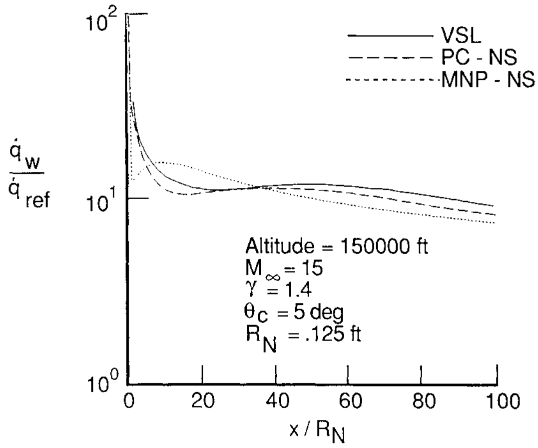


a) $\alpha = 3$ deg

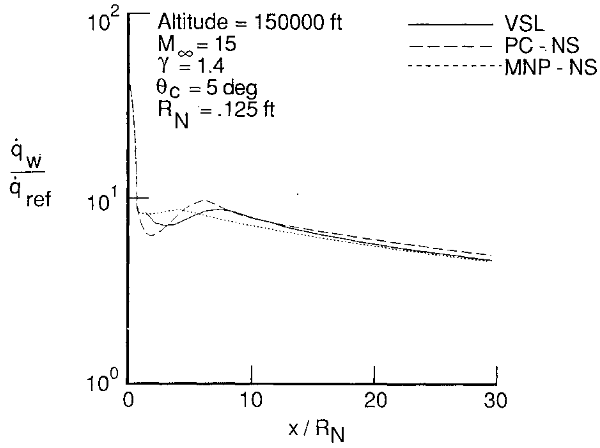


b) $\alpha = 20$ deg

Fig. 5 Influence of streamline calculation procedure on surface heating rates.



a) $\alpha = 3$ deg



b) $\alpha = 20$ deg

Fig. 6 Influence of pressure distribution on surface heating rates.

viscid pressure levels downstream of recompression are insignificant. However, in the overexpansion region, a discrepancy of approximately 10% is noted in the viscous and inviscid pressure levels.

The computed surface heating rates at 0-deg AOA (Fig. 4) are influenced only by the pressure distribution since the streamline metric reduces to the body radius in this case. The improved surface pressure correlations result in heating rates computed in the AEROHEAT code that are accurate within 10% of the VSL solution. The maximum error using modified Newtonian pressures is approximately 18%. Thus, for slender bodies, modified Newtonian pressures yield heating rates that compare poorly with more accurate solutions.

Heating rates for the blunted cone at AOA are shown in Fig. 5 for the simplified streamline (SS) method based on surface geometry and two methods for calculating streamlines from the pressure distribution. The pressures are computed with the present curve fits. One method for computing the streamlines and metrics bases the inviscid properties on variable-entropy edge (VE) conditions, and the second method uses inviscid surface properties based on the normal-shock (NS) entropy value. Heating rates computed with the pressure correlation and simplified streamlines are shown to be significantly lower ($\approx 40\%$) than the VSL results for the small AOA (3-deg) case; at the larger AOA (20 deg), the comparison is improved but significant discrepancies exist in a region just downstream of the tangency point. The heating results based on streamlines computed with pressures and variable-entropy edge conditions are shown to significantly underestimate the heating rates from the viscous-shock-layer solution. At the small AOA case (Fig. 5a), these results asymptotically approach the heating rates computed by the simplified streamline approach. At 20-deg AOA, the results of this heating method are shown (Fig. 5b) to yield very poor comparison with the VSL results. However, the heating rates computed using the pressure correlation and streamline calculation procedure based on inviscid surface properties (normal-shock entropy) predict the correct trends of the viscous-shock-layer solution with an accuracy within 10%. Therefore, it is clear that the present modifications in the pressure distribution and streamline geometry calculations greatly improve the resulting heating rates.

The influence of surface pressures on windward-ray heating rates is illustrated in Fig. 6. All heating rates are computed using the modified streamline calculation procedure based on inviscid surface properties (NS). Two pressure distributions are examined: the pressure correlations and modified Newtonian theory. Use of the inviscid pressure correlations at 3-deg AOA (Fig. 6a) results in approximate heating predictions that are in better agreement with VSL results than corresponding heating rates based on Newtonian pressures. Nevertheless, at 20-deg AOA (Fig. 6b), heating rates obtained using Newtonian pressures are fairly accurate, although the local minimum and maximum in the heating rate is not predicted. Newtonian theory does not account for the overexpansion of the pressure around the shoulder of a blunted cone nor the subsequent recompression on the conical afterbody and therefore cannot predict the corresponding local effects on the heating rates.

Concluding Remarks

An engineering code, AEROHEAT, was shown to yield poor comparisons in a recent investigation. Since this code is used throughout the aerospace community for heat-transfer studies, the deficiencies noted in that investigation need to be resolved. The major problems were related to the pressure and streamline calculation procedures.

The modified Newtonian pressure expression does not predict the extensive overexpansion/recompression region that occurs over blunt slender cones. A new pressure correlation and curve fit of detailed inviscid values for small cone angles was shown to yield improved heat-transfer comparison at 0-deg angle of attack. An existing pressure correlation was used for larger cone angles. For angle of attack, the rotated body approximation was used in the pressure correlations, and the resulting pressures were shown to be in good agreement with pressures calculated from a finite-difference Euler method.

At angle of attack, the calculations of the streamline and metric have a greater impact than the pressure calculation on the heating-rate level. Approximate heating rates computed from simplified streamlines based on surface geometry were shown to yield poor comparison with laminar heating rates computed by a three-dimensional viscous-shock-layer code for slender vehicles at small angles of attack. Likewise, poor comparison was obtained where the approximate heating rates were computed with streamline and metric calculations based on surface pressures and boundary-layer edge properties. Good agreement with detailed heating predictions was demonstrated when the AEROHEAT code employed the pressure correlations and used inviscid surface properties (normal-shock entropy) in the streamline and metric calculations. However, variable-entropy edge conditions are required in the heating-rate expressions.

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